Magnetic Field Evolution of Neutron Stars

Harshit Tiwari (21109866)* and Prof. M. K. Verma (Supervisor)†
Department of Physics, Indian Institute of Technology, Kanpur-208016, India
(Dated: November 11, 2021)

We did a literature survey around the astrophysics of neutron stars while focusing specifically on their magnetic field evolution. In this report, we discuss the structure and different relevant processes in neutron stars. The relevant model of neutron star is mentioned. The order of magnitudes of various quantities is calculated and mentioned. Neutron stars are highly magnetised and this magnetic field's evolution determines the evolution of neutron stars as a whole. We open the discussion for magnetic field evolution in isolated neutron stars, where we discuss the Ohmic dissipation and Hall drift in some depth. We also discussed some past results and the formalism of EMHD equations for finite conductivity. This report contains the basic formalism with proper referencing to be used for our future works. We also argue the applicability of this formalism in neutron star's crust. I also reviewed some important topics from hydrodynamics and magnetohydrodynamics (MHD), especially the energy transfers in fluid flows. We will use numerical simulations to solve the EMHD equations in the regime of neutron star's crust.

I. INTRODUCTION

Neutron stars (NSs) are compact objects which contain matter (presumably large fraction of neutrons) of superanuclear density in their interiors. As an endpoint of evolution of massive stars, they are (arguably) the only environment where extreme physical conditions of density, temperature, gravity and magnetic fields are simultaneously recognized.

Since they are formed when their progenitor's core reaches the **Chandrasekhar limit** (about $1.4M_\odot$), they are supported partially by neutron degeneracy pressure. A simple calculation of equating the neutron degeneracy pressure and gravitational pressure shows that the neutron degeneracy pressure is not by itself sufficient to hold up an object beyond $0.7M_\odot$. Thus, repulsive nuclear forces play a larger role in supporting more massive neutron stars.

They have typical masses $M \sim 1.4\ M_\odot$ and radii $R \sim 10$ km ($\sim 10^5$ times smaller than $R_\odot$). They have enormous gravitational energy, $E_{grav} \sim GM^2/R \sim 5 \times 10^{53}$ erg and surface gravity, $g \sim GM/R^2 \sim 2 \times 10^{14}$ cm s^{-2}. Their mean density, $\bar{\rho} \simeq 3M/(4\pi R^3) \simeq 7 \times 10^{14}$ g cm^{-3} is of the order of nuclear density ($\rho_0 = 2.8 \times 10^{14}$ g cm^{-3} is *normal nuclear density*).

Neutron stars are highly magnetised. Young neutron stars seen as ordinary radio pulsars and X-ray pulsars have their surface magnetic field deduced to be of order $\sim 10^{12} - 10^{13}$ G while older neutron stars are observed as recycled pulsars and low mass X-ray binaries have their surface fields $\gtrsim 10^{10}$ G [1]. Thus, their magnetic field evolve, decaying as they get old. There have been great advancements in understanding this evolution but it's still not understood well. Their high-energy radiation is non-thermal mostly, originated by particle acceleration (synchro-curvature emission) or Compton up-scattering of lower energy photons by the particles composing the magnetospheric plasma [5]. Both the isolated neutron star and accreting binary system rely on different physical understanding. Another class of objects consisting of anomalous X-ray pulsars and soft gamma repeaters are categorized under the term "magnetars" where magnetic field of underlying neutron star approaches $\sim 10^{15}$. Understanding the origin of the magnetic field in these exotic stars is an open problem in Physics right now.

This strong magnetic field of neutron stars is intimately coupled to the observed temperature and spectral properties and the observed timing properties (distribution of spin periods and period derivatives) [5]. Thus, taking into account the detailed calculations of microphysical properties (heat and electrical conductivity, neutrino emission rates) with a proper theoretical and numerical study of the magnetic field evolution equations is necessary. This study also helps in deciphering the physical processes behind the varied neutron star phenomenology.

This report contains a short overview of the structure of neutron stars while focusing mainly on the magnetic field evolution of **isolated** neutron stars. We write the basic equations and discuss in detail the nature of two terms in the induction equation: Ohmic dissipation and Hall term. This formalism is argued to be applicable in the magnetic field evolution in neutron star's crust.

II. NEUTRON STAR INTERNAL STRUCTURE AND PROCESSES

Neutron star is described by an *equation of state* (EOS) which often means the dependence of pressure P on the mass density ρ and on temperature T of matter. Since they are mainly composed of strongly degenerate fermions (neutrons, protons, electrons, and others), the temperature dependence is mostly negligible and the EOS can be calculated at $T = 0$, thus $P(\rho)$ only [3].

* tharshit@iitk.ac.in
† mkv@iitk.ac.in

The Fermi energy of neutrons in neutron star with nuclear density is calculated to be around 97 MeV which is around 10% of neutron rest mass, making them highly degenerate but still within non-relativistic regime. In contrast, the electrons inside a neutron star are highly relativistically degenerate with lower concentration (see Fig. 2.). According to current theories, a neutron star can be subdivided into the atmosphere and four main internal regions: the outer crust, the inner crust, the outer core, and the inner core as shown in Fig. 1. The *atmosphere* is a thin plasma layer, where the spectrum of thermal electromagnetic neutron star radiation is formed. The *outer crust* consists of heavy nuclei, in the form of either a fluid "ocean" or a solid lattice, and relativistic degenerate electrons. The matter of the *inner crust* consists of electrons, free neutrons, and neutron-rich atomic nuclei. The *outer core* consists of neutrons with several per cent admixture of protons, electrons, and possibly muons (the so called *npeμ* composition). Here, the neutrons are superfluid, with a smaller number of superfluid, superconducting protons and relativistic degenerate electrons. In the *inner core* there may or may not be a solid core consisting of pions or other sub-nuclear particles. The EOS of neutron stars cores is still a mystery while the relevant physics in their crusts has come to great understanding.

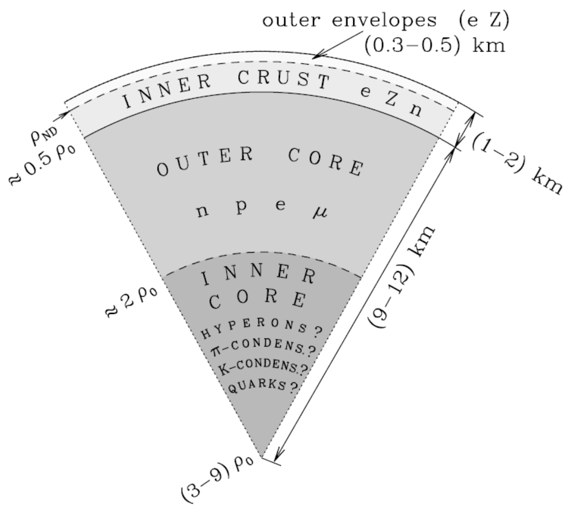

FIG. 1. Schematic structure of a neutron star. (Courtesy: Ref.[3]).

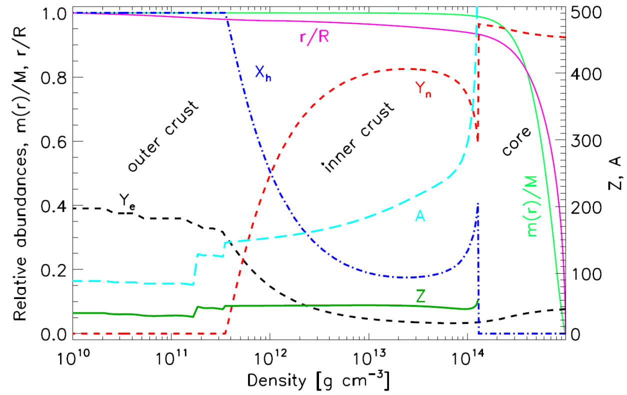

FIG. 2. Structure and composition of a $1.4M_\odot$NS, with SLy EoS.. (Courtesy: Ref.[3]).

Fig. 2. show a typical profile of a neutron star, obtained with the EoS SLy4 (Douchin and Haensel 2001) [3]. The plot shows, as a function of density from the outer crust to the core, the following quantities: mass fraction in the form of nuclei X_h, the fraction of electrons per baryon Y_e, the fraction of free neutrons per baryon Y_n, the atomic number Z, the mass number A, radius normalized to R, and the corresponding enclosed mass normalized to the star mass $m(r)/M$. For densities $\gtrsim 4 \times 10^{11}$ g cm^3, neutrons drip out the nuclei and, for low enough temperatures, they would become superfluid. Also the core contains about 99% of the mass and comprises 70–90% of the star volume (depending on the total mass and EoS).

Like white dwarfs, neutron stars obey a mass–volume relation, $MV = const.$, thus becoming smaller and more dense with increasing mass. There is also an upper limit on their mass, analogous to Chandrasekhar limit for white dwarfs, after which neutron degeneracy pressure and repulsive nuclear forces can no longer resist gravitational collapsing. This limit, called Tolman–Oppenheimer–Volkoff limit (or TOV limit) has a range of 2.2 to 2.9 $M_\odot$ after which the star collapse to form a black hole [2].

Since most stars cores rotate, the conservation of angular momentum shows that even a small rotation of the progenitor core can lead to the formation of rapidly rotating neutron stars with a rotation period on the order of few milliseconds. Using the conservation of magnetic flux, we can show trivially such high order of magnitude of magnetic field in neutron stars.

Neutron stars cools off via URCA process, $n \longrightarrow p^+ + e^- + \bar{\nu}_e$ and $p^+ + e^- \longrightarrow n + \nu_e$. They are extremely hot when they are formed during a supernova, $T \sim 10^{11}$ K. These neutrinos take away a large amount of energy, thus cooling the star to about 10^9 K in a day after the formation of neutron star and then slowly, due to other similar neutrino emitting processes (and later photon emitting), the surface temperature hover around 10^6 K for the next ten thousand years or so as the neutron star cools at an essentially constant radius [2]. Using Wien's displacement law, we can see that the radiation is primarily in the form of X-rays at $T = 10^6$ K.

The radial profile of electrical conductivity and magnetisation parameter $\omega_B \tau_e$ for three different temperatures is shown in Fig. 3. This was plotted using a modern Skyrme-type equation of state (EOS) at zero temperature, describing both the NS crust and the liquid core, based on the effective nuclear interaction SLy (Douchin Haensel 1991) [3]. Here, the NS model has a radius of

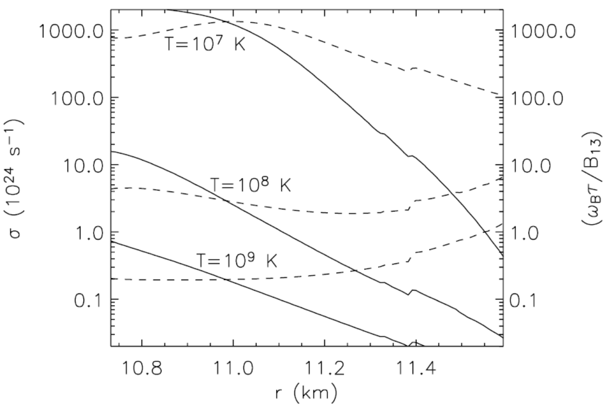

FIG. 3. Radial profile of electrical conductivity (solid line) and magnetisation parameter (dotted line). (Courtesy: Ref.[3]).

about 11.7 km and mass of $1.28M_{\odot}$. Its crust extends from 10.7 to 11.6 km. The figure demonstrates how the electric conductivity varies by 3-4 orders of magnitude within the crust and depends strongly on the temperature. The magnetization parameter scales linearly with B.

III. MAGNETIC FIELD EVOLUTION IN THE INTERIOR OF NEUTRON STARS

The interior of a neutron star is a complex multifluid system, where different species coexist and may have different average hydrodynamical velocities. We must rely on different levels of approximation that gradually incorporate the relevant physics.

The evolution of the magnetic field is given by Faraday's induction law:

$$\frac{\partial \boldsymbol{B}}{\partial t} = -c\nabla \times \boldsymbol{E} \tag{1}$$

which needs to be closed by writing the electric field $\boldsymbol{E}$ in terms of the other variables like the velocities of constituent component and the magnetic field itself. This is done by either using simplifying assumptions like Ohm's law or solving additional equations. Very often, neglecting the displacement currents, the electrical current density can be obtained from Ampere's Law

$$\boldsymbol{j} = \frac{c}{4\pi}\nabla \times \boldsymbol{B}. \tag{2}$$

In a complete multi-fluid description of plasmas, the set of hydrodynamic equations complements Faraday's law. From these multi-hydrodynamical equations, a generalized Ohm's law, where the electrical conductivity is a tensor can be derived by $\boldsymbol{j} = \hat{\sigma}\boldsymbol{E}$.

Goldreich and Reisenegger (1992) [1] proposed three basic processes responsible for magnetic field decay in isolated neutron stars. In the crust, the magnetic field evolves because of the Hall effect; essentially, the field is advected by the moving charges. In the deeper part of the crust and the core, where neutrons are abundant, ambipolar diffusion occurs, where the magnetic field and charged particles drift with respect to neutrons. Finally, Ohmic dissipation, due to finite conductivity, leads to the decay of the field in the crust. They obtained the electric field from the equations of motion of the charged particles, eq.(13) (from [1]) and thus getting an induction equation governing the magnetic field in eq.(15) (from [1]). This equation contains the various terms we talked about above.

In most of the crust, nuclei have very restricted mobility and form a solid lattice. Only the "electron fluid" can flow, providing the currents that sustain the magnetic field. Thus, in the crust, the electron velocity (which is also the fluid velocity) is simply proportional to the electric current

$$\boldsymbol{v}_e = -\frac{\boldsymbol{j}}{en_e}. \tag{3}$$

We take the electric field of the following form (generalised Ohm's law)

$$\boldsymbol{E} = \frac{\boldsymbol{j}}{\sigma} - \frac{\boldsymbol{v}_e}{c} \times \boldsymbol{B} \tag{4}$$

where the first term is simply the electric field in the reference frame comoving with matter, σ being the conductivity (temperature-dependent) taking into account the collision processes of charge carries (here electrons). The second term is due to the advection of the magnetic field lines by the charged component of the fluid, here electrons with velocity $\boldsymbol{v}_e$.

This form (4) of electric field do make sense for the electron momentum equation:

$$m_e n \left[\frac{\partial \boldsymbol{v}_e}{\partial t} + \boldsymbol{v}_e \cdot \nabla \boldsymbol{v}_e\right] = -\nabla p - ne\left[\boldsymbol{E} + \frac{\boldsymbol{v}_e \times \boldsymbol{B}}{c}\right] - \mu m_e n \boldsymbol{v}_e \tag{5}$$

where μ is the frictional coefficient having dimension of 1/[T]. Using (2), (3) and (4), the Hall-MHD (or electron-MHD) indcution equation reads:

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\frac{c}{4\pi}\nabla \times \left[\frac{\nabla \times \boldsymbol{B}}{en_e} \times \boldsymbol{B} + \frac{c}{\sigma}\nabla \times \boldsymbol{B}\right] \tag{6}$$

which can also be written as

$$\frac{\partial \boldsymbol{B}}{\partial t} = -\nabla \times \left[\eta\left(\nabla \times \boldsymbol{B} + R_m(\nabla \times \boldsymbol{B}) \times \hat{b}\right)\right]. \tag{7}$$

Here, $\eta = c^2/4\pi\sigma$ is the magnetic diffusivity, $R_m = cB/4\pi en_e\eta$ is the magnetic Reynolds number and $\hat{b}$ is the unit vector in the direction of magnetic field.

The magnetic diffusivity, η depends on temperature since the dependence of electrical conductivity σ on temperature. Hence, the magnetic and thermal evolution in the crust of neutron star are coupled and a proper study of magnetic field evolution of neutron star require both thermal and magnetic field evolution equations. The first term on the right hand side of (7) is responsible for Ohmic decay while the second term is the Hall term and its non-linear character is in the root of all difficulties we when numerical methods are used to solve the Hall induction equation. The magnetic Reynolds number R_m is an indicator of the relative importance between the Ohmic decay and Hall term.

The characteristic Ohmic dissipation timescale is

$$\tau_o = \frac{L^2}{\eta} \tag{8}$$

and Hall timescale is

$$\tau_H = \frac{4\pi e n_e L^2}{cB} = \tau_o / R_m \tag{9}$$

where L is the typical length scale. With these definitions, R_m is the ratio between the Ohmic dissipation and Hall timescales. Thus, the time-scale of the Hall effect depends on the strength of the magnetic field, the density of the crust and the magnetic field scale height.

Typical values of the electrical conductivity in the crust are $\sigma \sim 10^{22}$–10^{25} s^{-1} (which is several orders of magnitude larger than in the most conductive terrestrial metals). If we take the crust thickness ~ 1 km and number density $n_e \sim 10^{-35}$ cm^{-3}, we get a Hall time scale, $\tau_H \sim 60/B_{14}$ kyr, where B_{14} is the strength of the magnetic field in 10^{14} G units. While the Ohmic time scale is $\tau_o \sim 4$ Myrs (for $\sigma \sim 10^{24}$ s^{-1}).

Ohmic decay proceeds at rate inversely proportional to electrical conductivity and is independent of magnetic field strength. It occurs in both fluid core and solid crust of neutron star. In the core, the even larger electrical conductivity ($\sigma \sim 10^{26}$–10^{29} s^{-1}) results in much longer Ohmic timescales, thus potentially affecting the magnetic field evolution only at a very late stage ($t \sim 10^8$ yr), when isolated neutron stars are too cold to be observed [5].

The Hall drift speed is proportional to magnetic field strength and it occurs throughout the neutron star. Hall drift is not dissipative since it conserves magnetic energy and merely redistributes the magnetic field. From the above analysis, we can see that Ohmic evolution is two orders of magnitude slower than Hall. However, it can enhance the rate of Ohmic dissipation, thus indirectly responsible for magnetic field decay. This happens through the generation of smaller scale magnetic fields.

This was speculated by Goldreich and Reisenegger (1992) [1] where they showed that magnetic field undergoes a turbulent cascade terminated by Ohmic dissipation at small scales. Using the vector identity

$$\nabla \times (\nabla \times \boldsymbol{B}) = \nabla(\nabla \cdot \boldsymbol{B}) - \nabla^2 \boldsymbol{B} \tag{10}$$

and $\nabla \cdot \boldsymbol{B} = 0$, we can write (7) as

$$\frac{\partial \boldsymbol{b}}{\partial \tau} = -\nabla_\xi \times [(\nabla_\xi \times \boldsymbol{b}) \times \boldsymbol{b}] + \frac{\nabla_\xi^2 \boldsymbol{b}}{R_m} \tag{11}$$

where, $\xi = x/L$, $\boldsymbol{b} = \boldsymbol{B}/B_0$, and $\tau = t/t_h all$, with L and B_0 scale appropriate to the largest magnetic structures. This dimensionless equation resembles the vorticity equation for an incompressible fluid

$$\frac{\partial \boldsymbol{\omega}}{\partial \tau} = \nabla_\xi \times (\boldsymbol{v} \times \boldsymbol{\omega}) + \frac{\nabla_\xi^2 \boldsymbol{\omega}}{R_m} \tag{12}$$

and they argued that where $R_m \gg 1$ in the solid crust, the generic magnetic field evolves through a turbulent cascade.

For the case of perfectly conducting electron fluid, $\eta \longrightarrow 0$, the final EMHD equation is given by [4]

$$\frac{\partial \boldsymbol{Q}}{\partial t} + (\boldsymbol{v}_e \cdot \nabla)\boldsymbol{Q} = (\boldsymbol{Q} \cdot \nabla)\boldsymbol{v}_e + \mu d_e^2 \nabla^2 \boldsymbol{B} \tag{13}$$

where, $\boldsymbol{Q} = \boldsymbol{B} - d_e^2 \nabla^2 \boldsymbol{B}$ and

$$d_e^2 = \frac{c^2 m_e}{4\pi n e^2} = \frac{c^2}{\omega_e^2} \tag{14}$$

with ω_e^2 as the plasma frequency of electrons. Eq. (13) is actually the combined version of eq. (2), (5) and (7) for the perfectly conducting case. The phenomenology of EMHD turbulence of eq, (13) is presented for two regimes: $kd_e \ll 1$ and $kd_e \gg 1$ [4].

The energy spectrum in the $kd_e \ll 1$ regime is

$$E_Q(k) \approx E_B(k) = \frac{B_k^2}{k} = \epsilon_b^{2/3} k^{-7/3} \tag{15}$$

while for $kd_e \gg 1$

$$E(k) = \epsilon^{2/3} k^{-5/3} \tag{16}$$

To study a similar energy transfer for eq. (7), we need to numerically simulate it for the parameters (crustal density and conductivity profile) of neutron star. We talked about the neutron star model and the initial condition in II. Fig. 3. showed the magnetisation parameter, $\omega_B \tau_e$ which is equal to the magnetic Reynolds number R_m. We can also see the time scale of Hall and Ohmic dissipation term from this figure and to simulate neutron star crust, we have to take the dependence of both these parameters on temperature into account.

IV. FUTURE PLAN

Our goal in this semester is to understand the fundamental astrophysics of neutron stars while reading the essential topics, like their internal structure, in some depth. I simultaneously studied the important topics in magnetohydrodynamics (MHD), while also learning numerical techniques. We plan to solve the EMHD induction equation for finite conductivity in the regime of neutron star's crust and study the energy spectrum and other characteristics in different regimes.

[1] Peter Goldreich and Andreas Reisenegger, Magnetic Field Decay in Isolated Neutron Stars, 1992ApJ...395..250G.

[2] Bradley W. Carroll and Dale A. Ostlie, An Introduction to Modern Astrophysics.

[3] A.Y. Potekhin, D. G. Yakovlev, and P. Haensel, Neutron Stars 1: Equation of State and Structure.

[4] Verma, M. (2019). Energy Transfers in Fluid Flows: Multiscale and Spectral Perspectives. Cambridge: Cambridge University Press. doi:10.1017/9781316810019

[5] Pons, J.A., Viganò, D. Magnetic, thermal and rotational evolution of isolated neutron stars, Living Rev Comput Astrophys 5, 3 (2019).